Defending the Ramsey Test: What is wrong with Preservation?

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Abstract

In 'A Defence of the Ramsey Test', Richard Bradley makes a case for not concluding from the famous impossibility results regarding the Ramsey Test—the thesis that a rational agent believes a conditional if he would believe the consequent upon learning the antecedent—that the thesis is false. He lays the blame instead on one of the other premisses in these results, namely the Preservation condition. In this paper, we explore how this condition can be weakened by strengthening the notion of consistency which appears in it. After considering the effects of such weakenings for Bradley's argument, we propose a refinement of the Preservation condition which does not fall prey to Bradley's argument nor to Gärdenfors's impossibility theorem. We briefly compare it to Bradley's suggested restriction of Preservation.

In 'A Defence of the Ramsey Test', Richard Bradley (2007) makes a case for not concluding from the famous 'impossibility theorems' regarding the Ramsey Test—the thesis that a rational agent believes a conditional if he would believe the consequent upon learning the antecedent—that the thesis is false. He lays the blame on one of the other premisses of these theorems, the Preservation condition. This condition states that if one learns a sentence which is consistent with one's prior beliefs, then one retains all these prior beliefs after the revision. Bradley presents a series of arguments which purport to show that the Preservation condition, in the absence of the Ramsey Test, though along with other principles, leads to unacceptable results. Given that, as he argues, these other premisses do not seem to warrant excessive disdain, he concludes against the Preservation condition. Given the role of the Preservation condition in the classic arguments against the Ramsey Test, it follows that these arguments do not conclusively refute the Ramsey Test.

If one accepts Bradley's conclusion against the Preservation condition, as we shall for the purposes of this note, the following question remains: how much of the condition, in its traditional formulation, can be salvaged? An important point about the classical impossibility results and Bradley's arguments is that they involve the application of the Preservation condition to beliefs in conditionals. Hence one way to avoid the arguments is by denying that the Preservation condition holds for conditional sentences, whilst retaining it for factual sentences. This is what Bradley suggests.

Bradley is not the first to propose a solution of this sort. Isaac Levi has argued for a stronger position. He maintains that conditionals are not legitimate contents of belief (Levi 1988); it follows that the Preservation condition does not apply to them. Bradley is reluctant to go this far, preferring to motivate the restriction of the Preservation condition by the idea that conditional sentences, though legitimate objects of belief, are not the same types of sentence as factual sentences (Bradley 2007, pp. 11–14). If factual and conditional sentences are not of the same ilk, the argument goes, it is quite possible that some principles hold for the former but not for the latter. The Preservation condition is one of these.

This reply to the impossibility theorems may not be to everyone's taste. If you think that conditionals can be objects of belief and you do not embrace the nonfactualist view of conditionals (according to which conditionals do not bear truth-values), the restriction of the Preservation condition to factual sentences may seem to have an air of the ad hoc. Even if you do embrace non-factualism, the step leading to this restriction is not entirely clear: just because there is a difference between conditional and factual discourse as regards truth, why must there be a difference with regards to the application of the Preservation condition (which concerns belief)? For the sake of argument, and without wishing to take a position in the debate on the nature of conditionals, we shall assume for the rest of this paper that conditional sentences are legitimate objects of belief and we shall not assume any significant logical differences with respect to factual sentences. (In particular, following Bradley, we shall allow conditionals to be embedded under conditionals and other logical connectives.)

The aim of this note is to explore another way of weakening the Preservation condition which also avoids the impossibility results. The guiding idea is the following: given that the Preservation condition is a conditional statement—*if* one learns a sentence which is consistent with one's prior beliefs, *then* one retains all these prior beliefs after the revision—the condition will be weaker the stronger the antecedent is. Most notably, the antecedent features the notion of *consistency*. We will argue that the notion of consistency used traditionally in the literature is weaker than it should be, so that the antecedent is weaker that it should be, which in turn renders the condition stronger than it should be. By strengthening the relevant notion of consistency, we will propose a weakening of Preservation which resists the charge of impossibility.

After introducing the principles under discussion in section 1, we illustrate the importance of the notion of consistency adopted by showing that, just by mildly strengthening the notion of (logical) consistency adopted, Bradley's argument against Preservation can be rebutted (Sect. 2). In section 3, we propose a more appropriate version of the Preservation condition, arguing that we should employ in its antecedent not only logical consistency (non-contradiction), but also *epistemic* consistency (the possibility of coherently believing the new sentence in tandem with prior beliefs). The refined version of the Preservation condition will be shown to block the famous impossibility result (Sect. 4). Finally, we briefly compare the refined version of Preservation proposed here to the restriction proposed by Bradley (Sect. 5).

1 Impossibility results: a summary

The discussion will be couched in terms of the theory of Belief Revision; we shall not consider the probabilistic version of the Ramsey Test which, as Bradley notes, seems to pose a more daunting challenge. In this framework, one works with the language of classical propositional logic (connectives &, $\vee$, $\neg$, we will use $\supset$ for material implication), enriched with a binary connective $\rightarrow$ (the conditional). Call this language **L**. A consequence relation $\vdash$ containing classical propositional logic is assumed on the language. *Epistemic states* (denoted typically by K) are consistent sets of sentences of **L** closed under logical consequence. They are to be understood as representing the set of beliefs the subject has at a given moment. $\mathrm{K}^*_{\mathrm{A}}$ denotes the epistemic state obtained by revising K by a sentence $\mathrm{A} \in \mathbf{L}$. Following Gärdenfors (1986), we use the term *belief revision model* to denote the pair of a set of epistemic states and a function—the revision operation—taking pairs of epistemic states and sentences to epistemic states.

In this framework, the Ramsey Test is formulated as follows:

(RT) if $\mathrm{B} \in \mathrm{K}^*_{\mathrm{A}}$, then $\mathrm{A} \rightarrow \mathrm{B} \in \mathrm{K}$

Bradley (2007) distinguishes this from the other direction of the conditional, which he calls Conditional Driven Revision:

(CDR) if $\mathrm{A} \rightarrow \mathrm{B} \in \mathrm{K}$, then $\mathrm{B} \in \mathrm{K}^*_{\mathrm{A}}$

The Preservation condition is defined as follows:

(PRES) if $\neg \mathrm{A} \notin \mathrm{K}$ and $\mathrm{B} \in \mathrm{K}$, then $\mathrm{B} \in \mathrm{K}^*_{\mathrm{A}}$

In his seminal paper, Gärdenfors (1986) showed that there is no non-trivial belief revision model which satisfies (RT), (CDR), (PRES), and two other apparently harmless conditions. A belief revision model is said to be *non-trivial* if there are three mutually contradictory sentences A, B, and C and an epistemic state K such that each one of A, B, and C is consistent with K; we will say that such an epistemic state is a *non-trivial epistemic state*. The two other conditions in Gärdenfors's theorem, which shall be assumed throughout this paper, since they are also used in Bradley's argument, are the following:

(K^*_2) $A \in K^*_A$

(K^*_5) $K^*_A = K^*_\bot \Leftrightarrow \vdash \neg A$

Notice the role that the notion of logical consequence plays here. The consequence relation determines a notion of (logical) consistency of a set of sentences, or of a sentence with a given set of sentences. This notion of consistency appears in the definition of the epistemic states: being consistent according to and closed under the consequence relation $\vdash$, they are consistent according to the notion of consistency embodied by $\vdash$. So the very notion of epistemic state depends on the notion of consistency, and in particular on the consequence relation used: alter the notion of consistency—for example, by altering the consequence relation—and the notion of epistemic state will change. It follows that the contents of the principles, which all involve epistemic states, depend on the notion of consistency adopted. The Preservation condition is a particularly salient case in point, given that its antecedent basically involves a consistency condition: it demands that A is consistent with K according to the notion of consistency embodied by the consequence relation $\vdash$. Although we shall focus the discussion below on the Preservation condition, to facilitate comparison with Bradley's proposal, it could alternatively be formulated as a discussion of the appropriate notion of consistency for epistemic states.

In most papers on the subject, very little is said about the consequence relation: for example, Gärdenfors (1986) only requires that it contains classical propositional logic and satisfies the Deduction Theorem. Note however that it is not evident that the consequence relation of classical propositional logic is sufficient in the current framework, where the language contains the conditional connective $\rightarrow$. After all, demanding only classical propositional logic on this language amounts to assuming that the conditional $\rightarrow$ does not have any specific logical properties.

By contrast with Gärdenfors, Bradley does recognise that the conditional connective may have logical properties, and considers two in his paper. The first is a principle of conditional contradiction: for any set of sentences Δ,

(CC) if $\Delta, B, C \vdash \perp$ and $\Delta, A \nvdash \perp$, then $\Delta, A \rightarrow B, A \rightarrow C \vdash \perp$.

The second is a rule of Modus Ponens: for any set of sentences Δ and any A consistent with Δ,

(MP) if $\Delta \vdash A \rightarrow B$ and $\Delta \vdash A$, then $\Delta \vdash B$.

(We diverge slightly from Bradley's notation, using Δ instead of K to emphasise that, as the principles are stated, the sets of sentences are not assumed to be epistemic states. Moreover, the (CC) principle given above differs from that which appears in Bradley 2007, p. 5. The latter principle does not include the clause regarding the joint consistency of Δ and A, and is thus open to the objection that it is incompatible with several natural interpretations of $\rightarrow$. In particular, substituting $\bot$ for A and B, Bradley's original (CC) demands that $\Delta, \bot \rightarrow \bot, \bot \rightarrow C \vdash \bot$ for consistent Δ, and this is not an attractive condition.[1])

Bradley argues that the blame for Gärdenfors's impossibility result should be placed on the Preservation condition rather than on the Ramsey Test. To do so, he shows that, if the consequence relation satisfies these apparently natural principles of the logic of conditionals, (PRES) leads to unacceptable consequences.

Before moving on, let us note that the rule of Modus Ponens which Bradley uses is closely related to the following axiom of Modus Ponens, which is standard in the logic of conditionals (Stalnaker 1968, Lewis 1973):

(MP') $\vdash (A \rightarrow B) \supset (A \supset B)$

Given ordinary propositional logic, this is obviously stronger than (MP); however, in the presence of the Deduction Theorem, which Gärdenfors and other authors on the subject assume (Gärdenfors 1986, Rott 1989), they are equivalent. This is easily seen: from (MP), one can derive $A \rightarrow B, A \vdash B$ (by substituting $A \rightarrow B, A$ for Δ), which implies (MP'), in the presence of the

[1]Thanks to a referee for pointing out the modification which avoids this objection; according to this referee, Bradley has stated in private correspondence that the (CC) principle given above was what he intended.

Deduction Theorem.

2 Sensitivity to the notion of consistency

A central idea in this paper is that, by strengthening the notion of consistency, one can obtain natural weakenings of the Preservation condition which do not fall prey to the impossibility results. As noted, the consequence relation $\vdash$, which yields a notion of logical consistency, plays an important role in the definition of epistemic states, and in the formulation of the Preservation condition. The following fact illustrates the importance of the precise details of the notion of consistency used, and the power of the proposed method of refining Preservation: although Bradley's argument against (PRES) goes through if $\vdash$ satisfies only the tautologies of classical logic, (CC) and (MP), it is blocked as soon as $\vdash$ satisfies (MP') as well.

The argument with which we shall be concerned is the following (we paraphrase the argument in Bradley 2007, pp. 8–9). Suppose that the consequence relation $\vdash$ contains the classical consequence relation and satisfies (CC) and (MP), and that (PRES) holds. (Recall from section 1 that we are assuming (K^*_2) and (K^*_5) throughout this paper.) Let $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ be a set of mutually contradictory and jointly exhaustive sentences. Suppose that there is an epistemic state K containing none of A, ¬A, B, ¬B, C or ¬C, and both of the sentences $\mathrm{A} \vee \mathrm{B} \rightarrow (\neg\mathrm{A} \rightarrow \mathrm{B})$ and $\mathrm{A} \vee \mathrm{C} \rightarrow (\neg\mathrm{A} \rightarrow \mathrm{C})$. Henceforth, we will call epistemic states satisfying these conditions *B-states* (for 'Bradley-states'). Now consider the state K^*_A that results from modifying

K on learning or supposing that A. By (PRES), since A is consistent with K, both $A \vee B \to (\neg A \to B)$ and $A \vee C \to (\neg A \to C)$ belong to K^*_A. By (MP), it follows that both $\neg A \to B$ and $\neg A \to C$ belong to K^*_A. However, this violates (CC), since B and C are mutually contradictory and $\neg A$ is not logically contradictory (since it is consistent with K).

The argument applies the condition (PRES) to the revision of K by A, and thus depends on the assumption that K does not contain $\neg A$, although it contains the aforementioned conditionals. However, this assumption cannot hold if the consequence relation $\vdash$, in addition to satisfying (CC), also satisfies (MP'). Under these conditions, $A \vee B \to (\neg A \to B), A \vee C \to (\neg A \to C) \vdash \neg A$ when $\neg A$ is not logically contradictory. So, for non-contradictory $\neg A$, if K contains the two conditionals, then it contains $\neg A$; if K contains the two conditionals, then (PRES) does not apply. (If $\neg A$ is contradictory, the (CC) principle does not apply in the final step of the argument above, so the argument fails.)

To see that $A \vee B \to (\neg A \to B), A \vee C \to (\neg A \to C) \vdash \neg A$ when $\neg A$ is not logically contradictory, first note that, by (MP') and propositional logic, $(A \vee B) \to (\neg A \to B) \vdash A \supset (\neg A \to B)$ and $(A \vee C) \to (\neg A \to C) \vdash A \supset (\neg A \to C)$. So $(A \vee B) \to (\neg A \to B), (A \vee C) \to (\neg A \to C) \vdash A \supset ((\neg A \to B) \& (\neg A \to C))$ (by propositional logic). But, by (CC), and since $B, C \vdash \bot$ and $\neg A \nvdash \bot$, $(\neg A \to B) \& (\neg A \to C) \vdash \bot$; so $(A \vee B) \to (\neg A \to B), (A \vee C) \to (\neg A \to C) \vdash A \supset \bot$ (by propositional logic). Hence $(A \vee B) \to (\neg A \to B), (A \vee C) \to (\neg A \to C) \vdash \neg A$, as desired.

This, of course, may be taken as an objection to Bradley's argument.

Bradley considers it evident that there are B-states; his intention was to cast doubt on (PRES) by showing that, given the existence of a B-state and in the presence of (MP), (PRES) leads to a violation of (CC). But if the principle (MP') is accepted in the logic of conditionals, then the apparently mild assumption of his argument—that there exists a B-state—is inconsistent with the logic of conditionals; it follows that (PRES) cannot be applied as required. For his argument against Preservation to go through, Bradley must therefore defend a principle such as (MP) whilst rejecting (MP'). Although this position is doubtless not incoherent, it does not seem particularly natural: it is hard to see how one can argue that (MP) is acceptable while (MP'), which is obtainable from (MP) by a Deduction Theorem, is not.

As noted at the outset, our intention is not to challenge Bradley's position. To do that, it would be necessary to also consider the argument proposed later on in his paper, which does not involve Modus Ponens. Nor is it our intention to argue for any particular logic of conditionals. We concur with Bradley's conclusion that his argument 'leaves the project of finding a stronger interpretation of the conditional than the material conditional with a significant difficulty' (Bradley 2007, p9), although for slightly different reasons. The difficulty is that a theorist of conditionals who accepts (CC) must either deny that there are B-states, and Bradley gives a persuasive example to suggest that there are, or give up a *prima facie* intuitive principle of conditionals, (MP'). *Pace* Bradley, unless one can argue that (MP') and (MP) do not stand and fall together, giving up (PRES) does not free one from the dilemma.

Our intention in raising this point is simply to illustrate the importance of the notion of consistency for discussions about principles of belief revision. It shows that arguments involving these principles can be very sensitive to apparently minor alterations in the notion of consistency assumed, such as the shift from (MP) to (MP'). This is precisely the moral that will underlie the weakening of Preservation proposed in the next section. We will suggest that the traditional notion of consistency is inappropriate for epistemic states and languages including conditionals; a more appropriate, stronger notion leads to a weaker version of the Preservation condition, which does not fall prey to the impossibility arguments.

3 Weakening Preservation: a proposal

The motivating intuition behind the Preservation condition is the following: if a new piece of information is consistent with what was already believed, then no initial beliefs are given up on learning it. To cash out this intuition, a specific notion of consistency, and in particular consistency of beliefs, is needed. As noted in section 1, the formal rendering which has become standard, (PRES), assumes a logical notion of consistency, embodied by a consequence relation $\vdash$: consistency of a sentence A with an epistemic state K amounts to the fact that the two are not logically contradictory (according to $\vdash$). It is reasonable to question the assumption underlying this formalisation: is logical consistency the only sort of consistency which is relevant here?

If the underlying language containing the objects of potential beliefs is propositional, then, at least at the formal level, there seems to be no obstacle except logical contradiction to believing A and B simultaneously. However if the language contains conditionals, it is not so clear that this is still the case. Consider the following three sentences (where neither A and C are tautologous or contradictory): A, C, $\neg(C \rightarrow A)$. They are jointly consistent under the all of the logical principles considered above. Can one consistently believe the three sentences together? Certainly, if the conditional is read as a statement about, among other things, one's belief revision policy, as suggested by the intuition underlying the Ramsey Test, and if one does not change one's beliefs on learning something which one already believes,[2] the answer seems to be negative. For one would believe C, one would believe A, but one would also believe that it is not the case that, on coming to believe C, then one would believe A; this is, to say the least, an uncomfortable set of beliefs to have. For example, it would be consistent for John to simultaneously believe that it is raining, that the grass is getting wet, but also that it is not the case that if it rains, the grass gets wet.

The conjunction of the three sentences is in this sense like the famous Moore sentence ('it is raining but I do not believe it'): it is logically consistent but cannot be consistently believed. Indeed, this is more than a mere analogy. If one interprets conditionals along the lines suggested by the Ramsey Test, then, for someone who believes C, believing $\neg(C \rightarrow A)$ is tantamount to accepting that he would not believe A on learning what he already believes;

[2]More precisely, the principle alluded to here states that $K \subseteq K^*_A$ if $A \in K$.

unless one admits that one can retract beliefs on learning that which one already believes, this implies that he does not believe A. So believing A, C, and $\neg(C \rightarrow A)$ implies that one believes A and that one does not believe it.[3] In a word, although logically consistent, A, C, and $\neg(C \rightarrow A)$ are not *epistemically consistent*: they cannot coherently be simultaneously believed.[4]

If one accepts that the presence of a triple such as A, C, and $\neg(C \rightarrow A)$ constitutes a sort of inconsistency, then the condition (PRES) does not capture the intuition behind Preservation. In particular, it takes in its antecedent a notion of consistency which is too weak; hence the condition is stronger than it should be. A more appropriate principle would replace the clause '$\neg A \notin K$' in the antecedent of (PRES) by 'A is both logically and epistemically consistent with K', where the notion of epistemic consistency would be

[3] See also Levi 1988, Rott 1989, and Fuhrmann 1989, where is it is noted that the conditional operator can be used to say things about what the agent currently believes, but where different conclusions are drawn.

[4] Of course, this conclusion only holds if the conditional is read as stating something about beliefs, as a defender of the Ramsey Test would maintain. If, by contrast, conditionals are read as statements about causal relationships, say, then it may be possible to consistently believe a triple of the sort described. Consider for example someone who believes that it is raining, that London is the capital of the UK, but that it is not the case that, if it is raining, London is the capital of the UK. This seems acceptable if one reads the conditional as suggesting or affirming some causal link between the fact that it is raining and London's status in the UK; however, this triple of beliefs is less acceptable if the rejection of the conditional is taken to suggest that the person would no longer believe London to be the capital of the UK upon learning something which he already knew, namely that it is raining.

defined so as to rule out sets containing triples such as A, C, and $\neg(C \rightarrow A)$. This is tantamount to replacing an antecedent demanding logical consistency (the effective content of the clause '$A \notin K$', as noted in section 1) with one demanding both logical and epistemic consistency. In order to facilitate comparison with the Preservation condition as it is traditionally formulated, we shall assume for the purposes of this paper that the only cases of epistemic inconsistency which are not logically inconsistent are of the form given above, so as to obtain the following refined Preservation condition:

(PRES') if $\neg A \notin K$, for all $C \in K$, $\neg(C \rightarrow A) \notin K$, and $B \in K$ then, $B \in K^*_A$

We claim that this condition captures more accurately than (PRES) the intuition behind the Preservation condition in the case where the language contains conditionals.[5]

It is important that (PRES'), and the discussion motivating it, rests on the idea that there is a form of consistency that is relevant and that goes beyond logical consistency. By contrast with the logical principles considered in the previous sections, which are conditions on the consequence relation

[5] Of course, the condition that A be logically and epistemically consistent with K is not weaker than the condition in the antecedent of (PRES'), though it may be stronger. It follows that any refinement of the Preservation condition involving explicit reference to a primitive notion of epistemic consistency is weaker than the principle (PRES') that shall be used throughout the rest of this paper. Since the problem with (PRES) is that it is too strong, all of the points made below concerning the weakening obtained by replacing (PRES) by (PRES') continue to hold for other refinements based on the same idea, such as those involving explicit reference to a primitive notion of epistemic consistency.

(and, as a result, on the epistemic states), the notion of epistemic consistency only concerns beliefs, and does not apply to the logic. It is only being suggested that A, C, and $\neg(C \to A)$ cannot coherently be simultaneously believed, not that the three sentences are logically contradictory. Nor do the considerations above provide any reason for thinking that they are, just as the fact that we cannot believe 'it is raining but we do not believe it' does not imply that this sentence is a logical contradiction. An argument along the lines of the one presented in section 2—where, by replacing epistemic inconsistency by a principle according to which A, C, and $\neg(C \to A)$ are logically inconsistent, one blocks any conclusions drawn from (PRES')—is ineffective, because the grounds for accepting this epistemic refinement of Preservation do not justify the corresponding logical refinement.

4 Preservation and impossibility

Gärdenfors proved that there is no non-trivial belief revision model which satisfies (RT), (CDR), (PRES), (K_2^*), and (K_5^*); as noted above, he does not make any specific assumptions regarding the logic of conditionals. As Bradley notes (2007, p. 7), the role of (CDR) in Gärdenfors's result can be played by (CC). In fact, a Gärdenfors-like result can be obtained as a variant of Bradley's argument (presented in section 2). More precisely, let us call a consistent epistemic state which contains $\neg A \to B$ and $\neg A \to C$ for sentences A, B, C with $\neg A$ non-contradictory and B and C mutually contradictory, a *C-state* (for Contradiction-state). Bradley's argument shows, using (PRES), (MP), (K_2^*), and (K_5^*) that, given a B-state, one can obtain a C-state. Since, as he notes, the existence of C-states violates (CC), one obtains the desired contradiction. Similarly, the proof of Gärdenfors's theorem works by showing, using (RT), (PRES), (K_2^*), and (K_5^*), that given a non-trivial epistemic state, one can obtain a C-state. In traditional formulations (see Gärdenfors 1986 or Rott 1989), a contradiction is obtained from the existence of a C-state using (CDR) and (K_5^*); however, just as in the second part of Bradley's argument, one obtains the desired contradiction immediately from (CC). It follows that there are no non-trivial belief revision models satisfying (RT), (PRES), (CC), (K_2^*), and (K_5^*).

The C-state in Gärdenfors's argument is an expansion, where the expansion K_A^+ of an epistemic state K by a sentence $A \in \mathbf{L}$ is defined to be the set of $\vdash$-consequences of $K \cup \{A\}$.[6] The argument proceeds as follows. Let K be a non-trivial epistemic state, containing none of A, ¬A, B, ¬B, C or ¬C, with A, B, C mutually contradictory and jointly exhaustive. Since, by the definition of expansion, $A \notin K_{A \vee B}^+$, (PRES) applies to the revision of $K_{A \vee B}^+$ by ¬A. So $(A \vee B) \& \neg A = B \in (K_{A \vee B}^+)_{\neg A}^*$, and, by (RT), $\neg A \rightarrow B \in K_{A \vee B}^+$. Similarly, $\neg A \rightarrow C \in K_{A \vee C}^+$. Since $\neg A \notin K$, K_A^+ is consistent, and by the definition of expansion, $K_{A \vee B}^+$ and $K_{A \vee C}^+$ are subsets of K_A^+, so K_A^+ contains both $\neg A \rightarrow B$ and $\neg A \rightarrow C$. ¬A is not contradictory since $A \notin K$. K_A^+ is thus a C-state.

The weakening of Preservation which takes account of the possibility of

[6]For the sake of argument we assume, as Gärdenfors (1986, p. 85) does, that K_A^+ is always an epistemic state. See Rott 1989 for an examination of this assumption.

epistemic inconsistencies which are not logical inconsistencies blocks both Gärdenfors's and Bradley's arguments.

As concerns Gärdenfors's argument, if (PRES) is replaced by (PRES'), the passage from the existence of a non-trivial epistemic state to the existence of a C-state is no longer valid. With A, B, and C as above, let K be a consistent epistemic state containing none of A, ¬A, B, ¬B, C, and ¬C, but let it contain $\neg(A \vee B \to \neg A), \neg(A \vee B \to \neg B), \neg(A \vee C \to \neg A), \neg(A \vee C \to \neg C), \neg(B \vee C \to \neg C), \neg(B \vee C \to \neg B)$. K is thus a non-trivial epistemic state. The argument requires the application of (PRES) to the revision of $K^+_{A\vee B}$ by ¬A. Note however that, by the definition of expansion, both $A \vee B$ and $\neg(A \vee B \to \neg A)$ belong to $K^*_{A\vee B}$, so the antecedent of (PRES') is not satisfied if we substitute $K^+_{A\vee B}$ for K and ¬A for A. Hence (PRES') does not apply to the revision of $K^+_{A\vee B}$ by ¬A. Thus it does not follow that $(A \vee B)\&\neg A = B \in (K^+_{A\vee B})^*_{\neg A}$; similarly for the revision of $K^+_{A\vee C}$ by ¬A. However it is exactly these consequences of (PRES) that are required in the first step of the argument above. Since (PRES') does not apply here, the reasoning leading to the conclusion that K^+_A contains $\neg A \to B$ and $\neg A \to C$ is no longer sound. It has not been established that K^+_A is a C-state.[7]

Note that the step concerned here is common to both Gärdenfors's original theorem and the variant mentioned above where (CC) replaces (CDR). It follows that replacing the traditional Preservation condition (PRES) by

[7]In fact, the properties specified so far do not determine the value of $(K^+_{A\vee B})^*_{\neg A}$, so it can be specified so that it does not contain B; developing this, one obtains a belief revision model where K is a non-trivial epistemic state but where K^+_A is not a C-state.

the refined (PRES') blocks both of these versions of Gärdenfors's theorem; it is thus a way to avoid the famous argument that the Preservation condition and the Ramsey Test are inconsistent.

Moreover, (PRES') also provides an escape route from Bradley's argument; in this case, it blocks the passage from the existence of a B-state to the existence of a C-state. Let the consequence relation $\vdash$ satisfy (CC) and (MP) but not (MP'), and let A, B, C be as above. Let L be a consistent epistemic state containing none of A, $\neg$A, B, $\neg$B, C, and $\neg$C, but containing $(A \vee B) \rightarrow (\neg A \rightarrow B)$, $(A \vee C) \rightarrow (\neg A \rightarrow C)$, and $\neg(((A \vee B) \rightarrow (\neg A \rightarrow B)) \,\&\, ((A \vee C) \rightarrow (\neg A \rightarrow C)) \rightarrow A)$. So L is a B-state. However, (PRES') does not apply to the revision of L by A: the antecedent of (PRES') is not satisfied if we substitute L for K because of the presence of $((A \vee B) \rightarrow (\neg A \rightarrow B)) \,\&\, ((A \vee C) \rightarrow (\neg A \rightarrow C))$ and $\neg(((A \vee B) \rightarrow (\neg A \rightarrow B)) \,\&\, ((A \vee C) \rightarrow (\neg A \rightarrow C)) \rightarrow A)$ in L. But this is exactly the application of (PRES) which is required for Bradley's argument: it is needed to ensure that $(A \vee B) \rightarrow (\neg A \rightarrow B) \in L^*_A$ and $(A \vee C) \rightarrow (\neg A \rightarrow C) \in L^*_A$ (see Sect. 2). It has thus not been established that L^*_A is a C-state, and the contradiction with (CC) does not follow.

It is perhaps informative to translate the dry details of this argument into the terms of the example Bradley uses to motivate his claim that B-states exist (Bradley 2007, Ex. 4).[8] The example involves three urns, A, B, and C, one of which contains a prize. L is a B-state because it contains beliefs that,

[8] A similar exercise can be carried out for the epistemic state K proposed in the analysis of Gärdenfors's argument; it is omitted for lack of space.

if the prize is in A or B, then if it is not in A, then it is in B (and similarly for A and C); as Bradley notes, these are natural beliefs to hold in this situation. As for the negated conditional postulated in L, it is natural for anyone moved by Modus Ponens (either in the form (MP) or (MP')) and (CC). As shown by Bradley (see our Sect. 2), under Modus Ponens and (CC), one cannot simultaneously believe $(A \vee B) \to (\neg A \to B)$, $(A \vee C) \to (\neg A \to C)$, and A. It is thus natural to reject the conditional which says that if the first two hold, then the third holds: the postulate on L just states that it contains the negation of this conditional.

5 Towards a comparison

Richard Bradley has suggested replacing the Preservation condition by a restricted version, which applies only to sentences not containing the conditional connective: the 'B $\in$ K' in (PRES) is replaced with 'B $\in$ K not containing the connective $\to$'. Let us call this condition (PRES"). Both (PRES') and (PRES") escape Gärdenfors's impossibility result and Bradley's argument. In order to get a better idea of the significant issues on which these proposals differ, let us attempt a brief comparison on a simple example.

A philosophy position has been opened in a nearby university, and you are considering whether John will be selected. You have some information about his curriculum, and you believe that, apart from an excellent teaching record, it is decidedly unexceptional. Moreover, the head of the department has assured you that an excellent teaching record alone will not be sufficient to guarantee one the post. Nevertheless, you continue to suspend judgement about whether John will get the position or not. Three days later, you learn that the position is John's. How do you revise your beliefs?

Let C be the sentence 'John only has an excellent teaching record in his favour', and let A be the sentence 'John gets the position'. The advice of the department head, insofar as to pertains to John, can be formalised by the negated conditional $\neg(\mathrm{C} \rightarrow \mathrm{A})$: it is not the case that, if John only has excellent teaching credentials, then he will get the position. This is the translation which fits best with the Ramsey Test: the department head is suggesting that one should not come to believe that John will get the post (A) on learning that he only has excellent teaching credentials (C), and belief in $\neg(\mathrm{C} \rightarrow \mathrm{A})$ implies that one would not revise one's beliefs in this way.[9] So initially, you believe C and $\neg(\mathrm{C} \rightarrow \mathrm{A})$.

Learning that John gets the position (A) is a case where the original Preservation condition (PRES) applies. It implies that you should just add the belief that John has got the position to your original beliefs, retaining all prior beliefs and living with the tension among the beliefs that he only has teaching credentials (C), that teaching credentials alone do not suffice to

[9]Note the difference between formalising the statement by $\neg(\mathrm{C} \rightarrow \mathrm{A})$ and formalising it by $\mathrm{C} \rightarrow \neg\mathrm{A}$. In the former case, one may suspend judgement regarding John's success on learning that he only has teaching credentials. By contrast, in the latter case, one will come to believe that he will not get the post on learning that he only has teaching credentials (unless one violates (CDR)); as such, this latter formalisation risks being too strong.

guarantee him the post ($\neg$(C $\rightarrow$ A)) and that he has obtained the position (A); that is, you must live with the epistemic inconsistency. By contrast, the condition proposed above, (PRES'), does not apply, so this is a revision where some of the initial beliefs may be given up. In particular, to retain epistemic consistency on learning that he has obtained the position, you may relinquish either your conviction that his only credentials are in teaching (C) or your belief that these alone will not suffice to guarantee him the post ($\neg$(C $\rightarrow$ A)). None of the principles discussed in the previous sections would determine which belief to retract: that depends on the strengths of your beliefs, or more generally on your revision policy. Finally, Bradley's (PRES") does apply to this revision; it implies that in adding the new belief, you must retain your belief about John's teaching record (C), though you are allowed to retract your belief about the sufficiency of excellent teaching credentials for the success of the application ($\neg$(C $\rightarrow$ A)). If you wanted to retain epistemic consistency, (PRES") would thus force you to retract the latter, conditional belief. So, by contrast with (PRES'), Bradley's (PRES") does make a definite recommendation as to which of the prior beliefs to retract: put succinctly, if there is ever conflict between a piece of new information, prior beliefs in sentences not containing the conditional connective and prior beliefs in sentences containing the conditional connective, it will always be the latter which are given up.

It is far from clear that this is always a laudable recommendation. In the example, it could be that you are much more confident of your belief concerning the conditional, originating as it does from the head of department, than you are of your beliefs about John's qualifications. In such a case, it would not be irrational to retain your belief in the conditional and revise your belief about John's qualifications, even if this change is prohibited by the refinement of the Preservation condition proposed by Bradley. Of course, one could argue that the analysis above is naive, and that all beliefs concerning conditionals can be reduced to beliefs in factual sentences, so that there are factual sentences in the initial epistemic state which, in the presence of C, contradict A. In this case, (PRES") does not apply, and the conclusion above does not hold. However, there are powerful arguments against the reduction of beliefs in conditionals to beliefs in factual sentences, many of which have been offered by Bradley himself (Bradley 2005). Indeed, some have been offered as a consideration in favour of his refinement of Preservation (Bradley 2007, pp.13–14)!

In a nutshell, the difference between the two refinements of Preservation is as follows: whereas the refinement proposed here gives beliefs in conditionals equal rights in considerations regarding what is retained and what is retracted in a revision, the refinement proposed by Bradley treats conditionals as second-class citizens with respect to revision. This latter position only appears to be defendable if beliefs concerning conditionals are in fact second-class beliefs compared to beliefs in sentences not containing the conditional. If one thinks that this is indeed the case, as Levi for example does, then Bradley's refinement of Preservation seems to be the most natural option. If, by contrast, one does not think that this is the case, as Bradley appears to suggest, then the Preservation condition presented here seems to emerge as a more natural option for a defender of the Ramsey Test.[10]

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[10]The author wishes to thank Francesca Poggiolesi for helpful feedback on an early version of this paper, and two anonymous referees and the editor for their useful and detailed comments.

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